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IV. The diagonals of $ABCD$ and $EFGH$ meet in a point, V , the polar of $S'S''$ and therefore on the line RI (Booth, *New Geometrical Methods*, Vol. II., Art. 245). This includes Problem 187 of the ANALYST.

V. If $KLMN$ is the quadrilateral formed by joining the points of contact on the circle R , its sides and diagonals pass through the points already determined on the line $S'S''$; for N and K being centres of similitude of the circles A , D and R , NK passes through the external centre of similitude of A and D . The same is true for the corresponding circumscribed quadrilateral, $K'L'M'N'$, and the diagonals of these two quadrilaterals meet in a point, Q , which with the points $S'S^{iv}$ forms a triangle self-conjugate with respect to the circle R . In the same way it may be shown that $S'VS^{iv}$ is self-conjugate with respect to the circle I .

VI. The points S'' , S' , S''' , S^{iv} form an harmonic range (Catalan, *Théoremes et Problèmes*, III., 49).

VII. Since $BG = BL$ and $CG = CM$, L' is the centre of an exscribed circle to the triangle BRC and that circle is tangent to CB at G .

VIII. The circles about L' , M' , N' , K' as centres, and tangent to the sides of $ABCD$, form another set of four circles circumscribed about one, the circle O , and inscriptible in another, since the quadrilaterals $L'M'N'K'$ and $LMNK$ have the diagonals of each passing through the intersections of sides of the other, and are corresponding circumscribed and inscribed quadrilaterals to the circle R (Proposition III).

The points A , B , C , D are evidently on an ellipse to which the points R and I are foci. They are concyclic only when O and V (and therefore R and I) coincide, for only then is VS'' , or HF , perpendicular to EG .

IX. To inscribe in a given circle four circles tangent to each other and to another circle:—

Take any triangle, as $S'QS^{iv}$, self-conjugate with respect to the given circle, R . At K , L , M , N , the points where its sides cut the circle, R , draw tangents to R , and with their intersections as centres describe arcs orthogonal to R . From the points where the diagonals of $KLMN$ meet $S'S^{iv}$ draw tangents to these arcs; their intersections will be the centres of the required circles. The proof of this construction readily follows from the properties already obtained.

CUBIC EQUATIONS.

BY HENRY HEATON, SABULA, IOWA.

IF a and b be supposed to be either positive or negative, it is well known that every cubic equation may be reduced to the form

$$x^3 - 3ax = 2b. \quad (1)$$

Putting $x = \sqrt[3]{a}y$ and dividing by $a\sqrt[3]{a}$ we get

$$y^3 - 3y = \frac{2b}{a\sqrt[3]{a}} = 2c. \quad (2)$$

Putting $y = e^{\theta}\sqrt[3]{-1} + e^{-\theta}\sqrt[3]{-1} = 2\cos\theta$, this becomes

$$e^{3\theta}\sqrt[3]{-1} + e^{-3\theta}\sqrt[3]{-1} = 2\cos 3\theta = 2c. \quad (3)$$

$\therefore \theta = \frac{1}{3}\cos^{-1}c$, $y = 2\cos(\frac{1}{3}\cos^{-1}c)$, and $x = 2\sqrt[3]{a}\cos(\frac{1}{3}\cos^{-1}c)$,

a general expression for the roots of equation (1).

If we restrict $\cos^{-1}c$ to any one of its values the three roots are

$$\begin{aligned} x_1 &= 2\sqrt[3]{a}\cos(\frac{1}{3}\cos^{-1}c), \\ x_2 &= 2\sqrt[3]{a}\cos(\frac{2}{3}\pi + \frac{1}{3}\cos^{-1}c), \\ x_3 &= 2\sqrt[3]{a}\cos(\frac{4}{3}\pi + \frac{1}{3}\cos^{-1}c). \end{aligned}$$

If $a^3 > b^2$, c is less than unity and the three roots are real, and their values may be readily found from a table of cosines.

If $a^3 < b^2$, c is greater than unity and consequently $\cos^{-1}c$ is imaginary, yet the value of x_1 is real and might readily be found from such a table as the one given on p. 37, Vol. II, ANALYST, if it were sufficiently extended. Put $n = \cos(\frac{1}{3}\cos^{-1}c)$. Then is

$$\begin{aligned} x_1 &= 2n\sqrt[3]{a}, \\ x_2 &= -2\sqrt[3]{a} \cdot \left\{ \frac{1}{2}n + \frac{1}{2}\sqrt[3]{3(1-n^2)} \right\}, \\ x_3 &= -2\sqrt[3]{a} \cdot \left\{ \frac{1}{2}n - \frac{1}{2}\sqrt[3]{3(1-n^2)} \right\}. \end{aligned}$$

Since n is greater than unity, x_2 and x_3 are imaginary.

If a be negative c is imaginary and the expressions for the roots seem to be, but we shall see that one of them is real.

Because $\cos^{-1}c = \frac{1}{2}\pi - \sin^{-1}c$,

$$\begin{aligned} x_1 &= 2\sqrt[3]{a}\cos(\frac{1}{6}\pi - \frac{1}{3}\sin^{-1}c) = 2\sqrt[3]{a} \cdot [\frac{1}{2}\sqrt[3]{3}\cos(\frac{1}{3}\sin^{-1}c) + \frac{1}{2}\sin(\frac{1}{3}\sin^{-1}c)], \\ x_2 &= 2\sqrt[3]{a}\cos(\frac{5}{6}\pi - \frac{1}{3}\sin^{-1}c) = -2\sqrt[3]{a} \cdot [\frac{1}{2}\sqrt[3]{3}\cos(\frac{1}{3}\sin^{-1}c) - \frac{1}{2}\sin(\frac{1}{3}\sin^{-1}c)], \\ x_3 &= 2\sqrt[3]{a}\cos(\frac{3}{2}\pi - \frac{1}{3}\sin^{-1}c) = -2\sqrt[3]{a}\sin(\frac{1}{3}\sin^{-1}c). \end{aligned}$$

The expression for x_3 is real and its value might readily be obtained from the table above referred to.

SOLUTION OF AN INDETERMINATE PROBLEM.

BY DR. DAVID S. HART, STONINGTON, CONN.

To find general values of x and y to solve the problem

$$x^2 - Ay^2 = -1.$$

Let A be a non-quadrature number = the sum of two squares = $r^2 + s^2$,